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Rolling wing stability analysis

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Abstract

This report sets out to investigate the rolling moment due to rate of roll through experimental and theoretical methods. The significance of this derivative is laid out and its effects on real world applications is discussed. The experimental method involves measuring the roll rate due to an externally applied rolling moment on a straight tapered wing planform in a low speed wind tunnel. The theoretical analysis uses the Strip theory, modified Strip theory and Lifting line theory in order to attempt to estimate a value for L_p in wings of an elliptical and straight tapered planform. The results of this experiment both supported theoretical modelling but also disagreed with some assumptions made in the procedure. It has found a correlation between L_p and air flow speed and applied rolling moment, however the uncertainty due to error is high and clockwise-anticlockwise discrepancies throw doubt at the results.

Contents

Aims and objectives

This report aims to find the rolling moment due to rate of roll derivative (L_p) of a straight tapered planform through several different methods. The theoretical rolling moment due to rate of roll will be calculated using Strip theory, modified Strip theory and Lifting line theory for an elliptical wing planform, which will then be modified to calculate L_p through the same theoretical models but for a straight tapered wing. These results will then be compared to experimental data which will be calculated using a low-speed wind tunnel and a straight tapered planform. The objective being to verify the accuracy of the theoretical models as well as to investigate the effect of L_p on the stability of aircraft.

Background Theory

As a wing rolls, one side of the wing moves downwards increasing its incidence and the opposite side moves upwards decreasing its incidence. These forces act in the same rotational direction and provide an overall moment on the wing which acts in the direction opposing the motion of the wing. In practical applications this provides additional dynamic stability for an aircraft as the rolling moment due to rate of roll will dampen the rate of roll of the aircraft.

The rolling moment due to rate of roll for a wing can be found experimentally through the use of the rolling moment equation:

$$L = \frac{\frac{1}{2}\rho U_\infty^2 S_W 2s L_p (p2s)}{U_\infty} \quad [1]$$

This rearranges to give the rolling moment due to rate of roll L_p:

$$L_p = \frac{\frac{L}{p}}{\frac{1}{2}\rho U_\infty S_W 4s^2} \quad [2]$$

U_∞ = Wind speed in tunnel

S_W = Surface area of wing

s = Span of half of the wing

ρ = Density of air at test conditions

The theoretical rolling moment due to rate of roll will be calculated using Strip theory, Modified Strip theory and Lifting line theory, as defined below.

For elliptical wings:

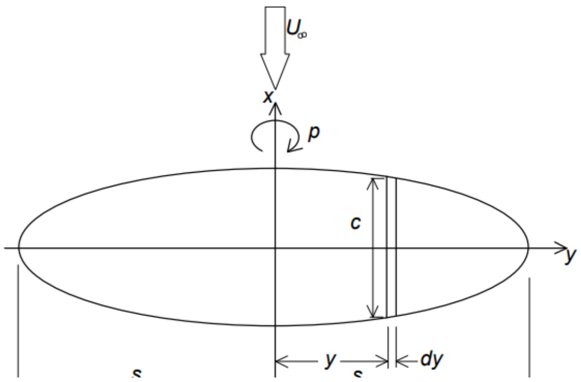

Figure 1: Diagram of an elliptical wing

The incidence change due to rolling at an angular rate of p is given bellow with an approximation for moderate roll rates

$$\tan^{-1}(\frac{py}{U_\infty}) \approx \frac{\text{py}}{U_\infty}$$

[3]

For a lift-curve slope of a_∞ the change in the lift is given by

$$a_\infty \frac{py}{U_\infty}\frac{1}{2}\rho U_\infty^2 c \text{ dy}$$

[4]

For a rolling moment of L about the x is

$$L = -\frac{1}{2}\rho U_\infty p a_\infty \int_{-s}^{s} \text{cy}^2 \text{ dy}$$

[5]

For an elliptical wing

$$C_l = -\frac{a_\infty \bar{p}}{16} \text{ where } \bar{p} = \frac{p2s}{U_\infty}$$

[6]

Therefore:

$$L_p = -\frac{a_\infty}{16}$$

[7]

Modified Strip theory

In order to compensate for effects due to trailing vortexes, the aspect ratio of the wing can be taken into account giving:

$$L_p = -\frac{a_\infty}{16(1 + \frac{a_\infty}{\pi \text{ AR}})}$$

[8]

Lifting line theory

Due to the loading on the wings not being symmetrical, the modified strip theory can be adjusted for a fuller analysis giving:

$$L_p = -\frac{a_\infty}{16(1 + \frac{2a_\infty}{\pi \text{ AR}})}$$

[9]

The theoretical modelling using Strip and Lifting line theories can be modified to better suite the experiment carried out in the report by adjusting for the straight tapered wing used.

From equations 5 and 6, For straight tapered wing:

$$L_p = -\frac{a_\infty \int_{-s}^{s} cy^2 dy}{(2s)^2 S_W}$$

[10]

$$where \int_{-s}^{s} cy^2 dy = 2\int_{0}^{s} cy^2 dy$$

$$and\ C = C_0 + \frac{y}{s}\,(C_t - C_0)$$

$$therefore \int_{-s}^{s} cy^2 dy = 2C_o \frac{s^3}{3} + \frac{(C_t - C_0)s^3}{2}$$

[11]

From 10 and 11, K the straight tapered wing correction can be found as

$$K = \frac{(2s)^2 S_W}{2C_o \frac{s^3}{3} + \frac{(C_t - C_0)s^3}{2}}$$

[12]

Taking the original theoretical solutions for a straight tapered wing, the 16 in the denominator can be replaced by the straight tapered wing correction K to give:

Strip theory

$$L_p = -\frac{a_\infty}{K}$$

[13]

Modified Strip theory

$$L_p = -\frac{a_\infty}{K(1 + \frac{a_\infty}{\pi\ \mathrm{AR}})}$$

[14]

Lifting Line theory

$$L_p = -\frac{a_\infty}{K(1 + \frac{2a_\infty}{\pi\ \mathrm{AR}})}$$

[15]

Apparatus and Instrumentation

A straight tapered wing was placed in the centre of a low speed wind tunnel with the front of the wing as the leading edge facing the oncoming flow. A diagram of the wing used is given below in figure 2 with dimensions in figure 3.

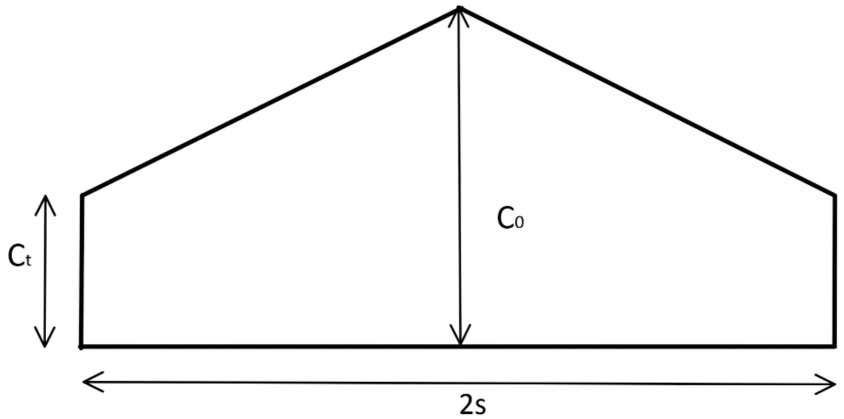

Figure 2: Tapered wing diagram

C_t	0.062cm		C_0	0.123cm		2s:	0.515cm	Surface area:	0.047638 m^2

Figure 3: Wing data

The wing is mounted on a shaft attached using bearings (Figure 4) in order to be freely rotating with minimal friction. A gear is attached to the supporting shaft with a string tied around (Figure 5) it in order to provide an external rolling moment to the wing. The other end of the string is attached to a 0.5Kg plate which is able to hold extra weight plates in increments of 0.5Kg. Additionally, a laser activated timer is connected to the gear which uses two holes in the profile of the gear in order to count the time taken for 10 rotations. A betz manometer is used to measure the dynamic pressure in the tunnel.

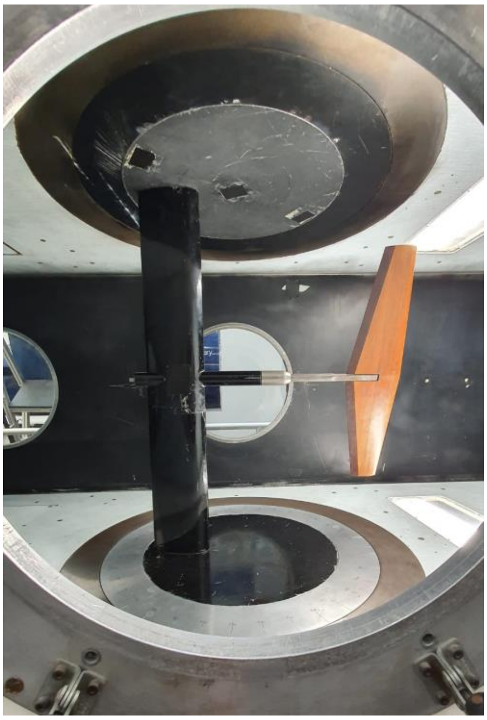

Figure 5: Straight Tapered wing placed in the wind tunnel

Figure 4: Gear and string suspending weights

Procedure

1. The room conditions are measured. Atmospheric Pressure (Analog Barometer), Temperature (Analog thermometer), and the wing data (Tape measure) is taken. The length of the string for 10 revolutions and the thickness of the weight plate are measured (Tape measure)
2. Automatic timer is reset, and the wind tunnel is started at the first reference pressure of 15mmH2O in the betz manometer.
3. The weights are released from rest and travel until they reach the floor, the timer automatically counts and times 10 revolutions.
4. The gear is wound up again and reset to starting conditions
5. Steps 2 to 4 are repeated for reference pressures of 15,20,25mmH2O in increments of 0.5Kg from 1 to 2.5Kg and for both clockwise and counter clockwise rotation in all cases.

Raw data

The raw data collected from the carried-out experiment has been tabulated as shown below:

Bobbin Radius: (m)		0.010488		
H1:	0.009		H2:	0.668

Figure 6: Measured data for Bobbin Radius

Temp:	18.5	C	Atmospheric Pressure:	100200	Pa	Density:	1.196665
	291.65	K					

Figure 7: Atmospheric data

		Wind Speed (mmH2O)	15	20	25
Time (s)	1Kg	Clockwise (s)	17.34	18.15	19.39
		Counter- (s)	21.11	23.18	24.34
	1.5Kg	Clockwise (s)	11.29	12.14	13.11
		Counter- (s)	12.19	14.31	15.85
	2Kg	Clockwise (s)	8.16	9.09	10.29
		Counter- (s)	9.01	10.80	11.07
	2.5Kg	Clockwise (s)	6.25	7.18	8.24
		Counter- (s)	7.00	8.12	9.03

Figure 8: Data Measured

Calculations

From equation 1, the equation for $\frac{L}{p}$ which can also be found from the gradient of L against p graph.

$$\frac{L}{p} = \frac{1}{2}\rho U_\infty S_W 2sL_p$$

[16]

Density can be found through

$$\rho = \frac{P}{RT}$$

$$\rho = \frac{100200}{287 * 291.65} = \ 1.196665\frac{Kg}{m^3}$$

Wind tunnel speed can be found from

$$U_\infty = \sqrt{\frac{(K * \rho_w g h)}{\frac{1}{2}\rho}} = \sqrt{\frac{(1.03 * 147.1)}{\frac{1}{2} * 1.1967}} = 15.91\,\frac{m}{s}$$

Tunnel velocities are calculated as above

mmH2O	15	20	25
Pa	147.0998	196.133	245.1663
U_∞ (m/s)	15.91305	18.3748	20.54365

Figure 9: Tunnel wind speeds

Wing area can be calculated by assuming the wing as a trapezium and using

$$s(C_t + C_0)$$

Taking data from figures 6 and 3:

$$S_W = 0.2575(0.062 + 0.123) = 0.0476375\, m^2$$

Rate of roll of the wing can be taken from the time for 10 revolutions

For 1Kg at 15.91m/s, t = 17.34s for 10 revolutions

$$\frac{t}{revs} = 1.734\,\frac{s}{2\pi}$$

$$\frac{rad}{s} = \frac{1}{1.734} * 2\pi = 3.62$$

Taking data from figure 6, the rolling moment can be calculated by

$$L = mgr \quad where\ r = \frac{H_2 - H_1}{20\pi}$$

The rolling moment against rate of roll for all tests is shown below

	L (Nm)	Wind Speed m/s	15.91305	18.3748	20.54365
Roll rate (rad/s)	0.102785	Clockwise	3.62	3.46	3.24
		Counter-	2.98	2.71	2.58
	0.154178	Clockwise	5.57	5.18	4.79
		Counter-	5.15	4.39	3.96
	0.205571	Clockwise	7.70	6.91	6.11
		Counter-	6.97	5.82	5.68
	0.256964	Clockwise	10.05	8.75	7.63
		Counter-	8.98	7.74	6.96

Figure 10: Rolling moment against Rolling rate data

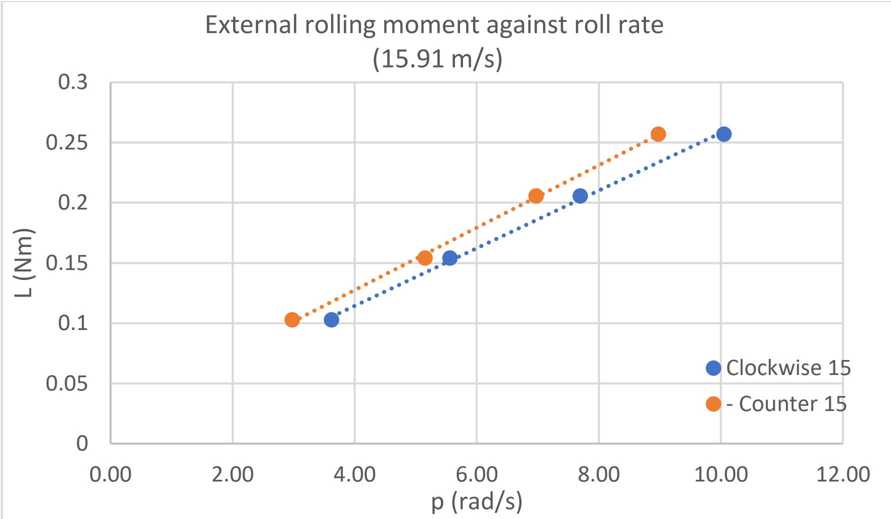

Figure 11: Rolling moment against roll rate (15.91 m/s)

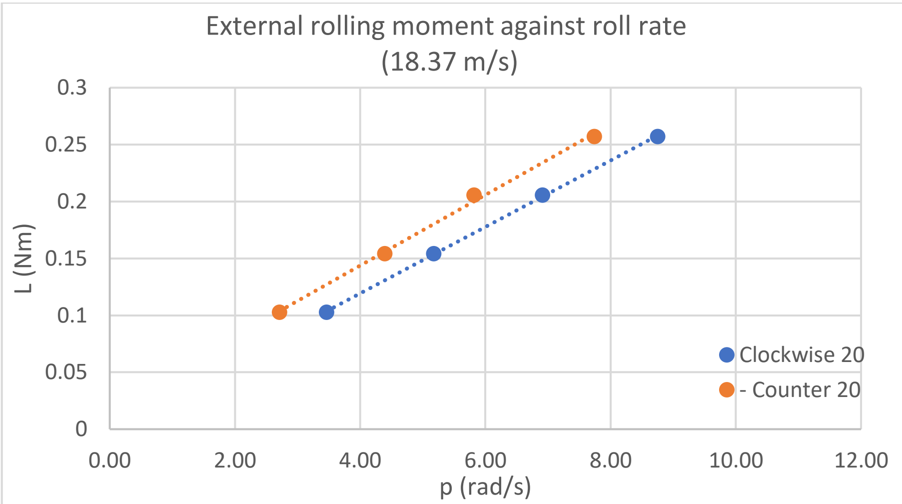

Figure 12: Rolling moment against roll rate (18.37 m/s)

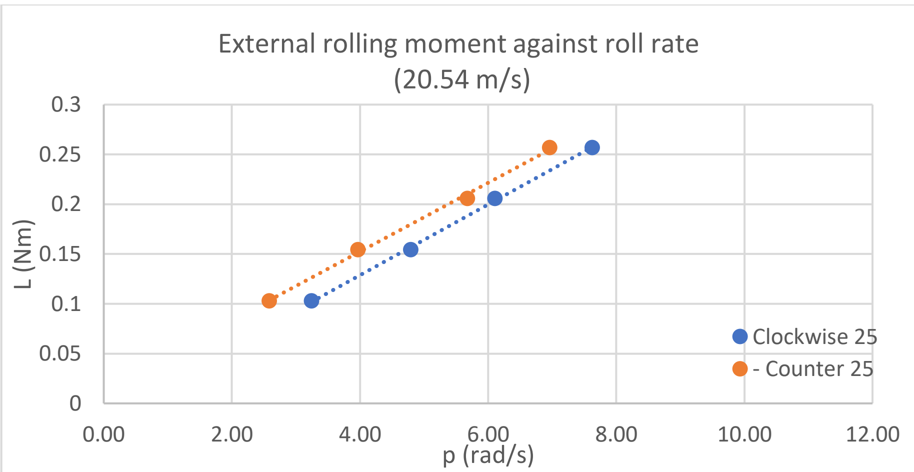

Figure 13: Rolling moment against roll rate (20.54 m/s)

Reynolds number calculations

$$Re = \frac{\rho U_{\infty} \bar{C}}{\mu} \quad where, \bar{C} = \frac{S_W}{2s} = 0.0925, and\ \mu = 1.8e^{-5}$$

$$Re = U_{\infty} * 6.14e^{3}$$

U_{∞}	Re
15.91304546	9.77E+04
18.37480216	1.13E+05
20.54365335	1.26E+05

Figure 14: Reynolds numbers corresponding to wind tunnel speeds

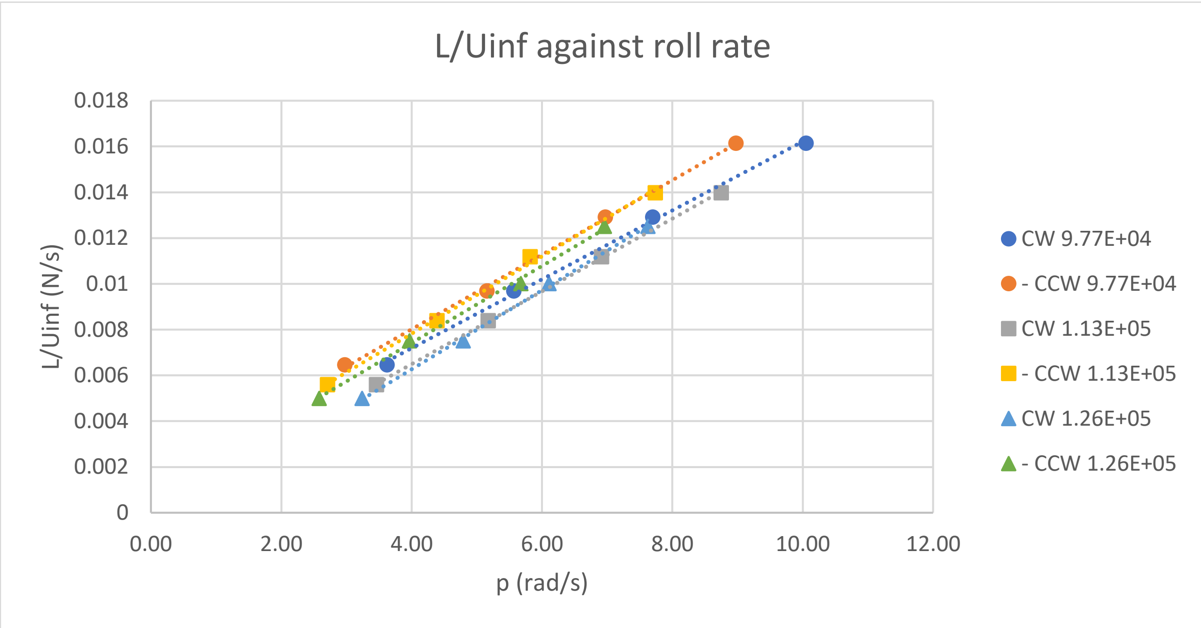

Figure 15: L/Uinf against roll rate at values of Re from Figure 14

Calculation of angle of attack

$$Angle\ of\ attack = \tan^{-1}(\frac{py}{U_{\infty}}), where$$

p is rolling moment when graph begins to diverge from line

	AOA (°)	
Air speed (m/s)	clockwise	counter
15.91305	9.239717	8.264221
18.3748	6.991487	6.188814
20.54365	5.45956	4.984445

Figure 16 AoA at stall for corresponding air speeds

Standard deviation calculations from population in figure 10

L_p mean = 0.216346329

Deviations	Square of deviations
-0.017673431	0.00031235
-0.001048084	1.09848E-06
-0.006135924	3.76496E-05
0.006822251	4.65431E-05
0.012237079	0.000149746
0.00579811	3.36181E-05

Sum of Squared Deviations	Variance	Standard deviation
0.000581006	9.68343E-05	0.00984044

Results

$$L_p = -0.2163 \pm 0.0098$$

Theoretical L_p can be calculated for an elliptical wing from equations 7,8,9 and for a straight tapered wing from equations 13,14,15 in the background and theory section.

Theoretical Elliptical results		
a_∞	5.7	6.283185
Strip Lp	-0.35625	-0.3927
Modified Strip Lp	-0.26869	-0.28891
Lifting Line Lp	-0.21568	-0.22852

Figure 17: Theoretical Lp for an elliptical planform

Theoretical Straight Tapered results		
a_∞	5.7	6.283185
Strip Lp	-0.39669	-0.43728
Modified Strip Lp	-0.29919	-0.32171
Lifting Line Lp	-0.24016	-0.25446

Figure 18: Theoretical Lp for a straight tapered planform

Discussion

By looking through graphs in Figures 11 through 14 it can immediately be seen that a higher velocity of air flow, corresponding to a higher Reynolds number, over the experimental straight tapered planform leads to a greater damping effect experienced by the planform. This can be seen as the maximum rate of roll decreases almost linearly with the increase in air flow velocity, going from ~ 10 rad/s to under 8 rad/s from 15.9 to 20.5 m/s of air flow. This corresponds to an average decrease of 1.44 rad/s per 2.32 m/s increase in flow speed showing a negative correlation between the two variables.

Looking at the graph of Figure 15 it can be seen that the results support the point that the rolling moment due to rate of roll aids in the rolling stability of a wing as it shows that the higher the applied rolling moment to air flow speed loading leads to a larger damping force which can be seen as the overall gradient of the graph remains linear, until it approaches its stall conditions. This means that larger external forces such as a gust or weight shift in an aircraft will receive a proportionally larger damping force which will further aid in the aircraft’s stability. Had the rolling moment due to rolling only been dependent on the wing profile and not the loading or air flow speed, then the damping would be constant and not sufficient for large loads.

Taking the percentage error from the theoretical data as being

$$error = \frac{theoretical - experimental}{experimental} * 100$$

The error in the theoretical assumptions can be seen in Figure 19 below

	Elliptical planform		Straight Tapered planform	
	Error (%)		Error (%)	
a_∞	5.7 rad	2π rad	5.7 rad	2π rad
Strip Theory	64.66653	81.5141	83.3584105	102.1183984
Modified Strip theory	24.19402	33.54251	38.29172013	48.7013931
Lifting Line theory	0.3086	5.626847	11.00772154	17.61692109

Figure 19: Errors for theoretical results as compared to experimental

Figure 19 has been tabulated from taking an experimental value of -0.2163 and finding the percentage error for each theoretical value. It can be seen that there is a large variance in the results from each of the theoretical models used even when comparing between the elliptical and straight tapered planform. Figure 19 shows a larger agreement between the elliptical planform over the straight tapered planform assumption with the experimental data. The data converges towards the experimental value for both wing planforms as the complexity of the theoretical model increases, with the lifting line theory of an elliptical planform at a value of a_∞= 5.7 rad.

When looking at Figures 11 through 14 for validity, it can not be seen if the data has reached stall conditions therefore a definitive assumption of stall conditions can not be made. Furthermore, the graph shows that the clockwise and anticlockwise values of L/p are different however these are expected to be of the same magnitude, this suggests that there are possible systematic errors in the experiment done, which will be further discussed in the report.

<u>Errors</u>

As mentioned in the discussion the analysis of the experimental results of L/p suggest that there has been a discrepancy in the results expected within the experimental modelling. These errors can be further broken down into the systematic and human errors which were likely to have occurred during the procedure.

Systematic- While there may have not been one largely significant error that took place, the apparatus and instrumentation used lead to a significant build up in numerous areas. Of these the most significant error would be the discrepancy in the values obtained from clockwise and anticlockwise rotations, which can be down to the overall swirling occurring in the wind tunnel due to the fan creating the pressure difference in the tunnel. This is multiplied by the fact that the working area of the tunnel was not significantly large enough and the planform took up a large proportion of the working area. Another area of error would be due to the friction in the shaft holding the planform in place, which would reduce the applied rolling moment on the planform.

Human error- As with all procedures requiring human input, the results of this report are swayed by the accuracy at which measurements were taken. As all measurements were taken in an analog reading meaning that both the accuracy of readings could be low due to low experience of the group, as well as the low precision due to the tape measure.

Difficulties and improvements

As previously stated, there were many aspects in the procedure, both systematic and human, which could have produced an error in results. The following could be done to reduce these:

1. Using a wind tunnel with a larger working section
2. Using a digital thermometer and barometer
3. Using a tape measure with a higher degree of precision
4. Taking more experimental data in order to find when stall begins

Conclusion

This experiment showed that the rolling moment due to rate of roll is a crucial element in the rolling stability of aircraft as it provides a damping force which is proportional to the applied rolling moment and air flow velocity. The results of theoretical and experimental data have also been compared with the greatest agreement being with the lifting line theory of an elliptical planform at a value of a_∞= 5.7 rad. While these conclusions have been drawn to the data the experiment has not been completely successful as the systematic errors in the experiment and lack of quantity of experimental data make the results questionable. In order for the data to be verified, more testing must be done in a larger wind tunnel.

References

SC1 Rolling moment due to rate of roll, department of engineering, Queen Mary University of London, 2013